Talk: PACT V - Khovanov homology of slice disks

September 9, 2021 1:35 PM

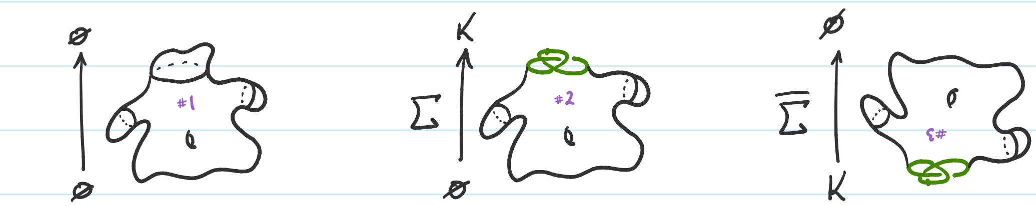
I. MOTIVATION

II. KHOVANOV HOMOLOGY OF LINKS III. AND SURFACES

IV. KHOVANOV-JACOBSSON CLASSES

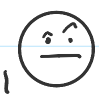
V. KHOVANOV HOMOLOGY OF SLICE DISKS

We consider 3 cases:



Aside: Link cobordisms can be composed! Given $X: L_0 \to L_1$ and $Y: L_1 \to L_2$, we can form $Y \circ X: L_0 \to L_2$ by stacking Y onto X.

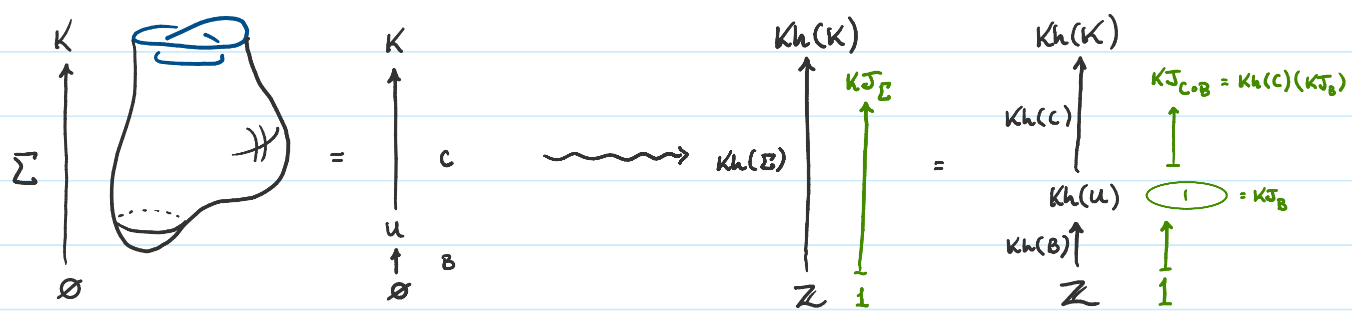
Note: Case #1 looks a lot like #2 with something stacked on it!



THOUGHT EXPERIMENT

Suppose $\Sigma: \emptyset \to K$ is a ribbon disk. Then $\Sigma = C \circ B$, where $C: \mathcal{U} \to K$ is a ribbon-concordance and $B: \emptyset \to \mathcal{U}$ is a Morse birth.

unknot



Recall ribbon-concordances induce injections and 1 "KJ_B" is a nontrivial class in $Kh^{0,0}(\mathcal{U})$ because it is not a boundary (note $Kh^{-1,0}(\mathcal{U})$ is trivial).

Thus $KJ_\Sigma = KJ_{C\circ B}$ is nontrivial!

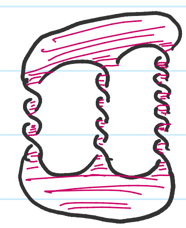
! If we find a slice disk $D:\emptyset \to K$ with trivial KJ_D then K cannot be a ribbon knot (disproving the Slice-Ribbon conjecture!)

Thm ★ (Swann '10) For a link cob. $\Sigma : \emptyset \to K$ with genus $g(\Sigma) \leq 1$, if K is slice, then KJ_Σ is nontrivial.

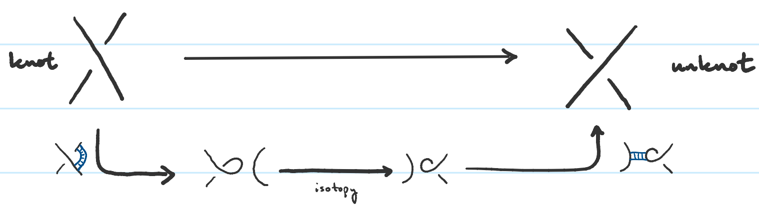
- $g=0 \Rightarrow$ slice disks also have nontrivial KJ-classes
- $g=1 \Rightarrow$ KJ-classes can obstruct sliceness (of knots w/ 4-ball genus at most 1)

Thm (Swann '10) Odd 3-stranded pretzel knots $P(a,b,c)$ with a,b,c all positive bound genus 1 Seifert surfaces with trivial KJ-classes, and thus, are not slice.

$P(3,5,7)$

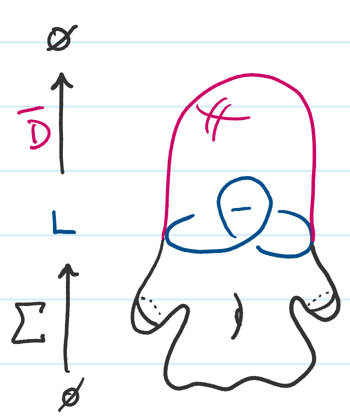


Note: Knots w/ unknotting number 1 bound genus 1 surfaces and are thus good candidates



Ex's Conway knot, Whitehead doubles

Proof of ★ Let $g(\Sigma)=1$ and assume K slice. ($\exists D:\emptyset \to K$) Consider $\bar{D}\circ\Sigma : \emptyset \to \emptyset$



$$\begin{array}{ccc} \mathbb{Z} & & 2 \\ \uparrow & & \uparrow \\ Kh(K) & & KJ_\Sigma \\ \uparrow & & \uparrow \\ \mathbb{Z} & & 1 \end{array}$$

Case #1

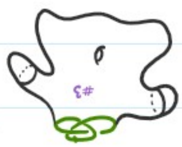
if $KJ_\Sigma = 0$ then it can't be sent to 2 by a homomorphism

$\Rightarrow KJ_\Sigma \neq 0$

For $g(\Sigma)=0$ consider $(\overline{D} \# T^2) \circ D : \emptyset \to \emptyset$

<u>Case #3</u> joint w/ Kyle Hayden (HC '13)

$K \xrightarrow{\overline{\Sigma}} \emptyset$



$\rightsquigarrow$ $\mathrm{Kh}(K) \xrightarrow{\mathrm{Kh}(\overline{\Sigma})} \mathbb{Z}$ $\qquad \phi \mapsto \Sigma_\phi$

<u>IDEA</u> Reverse the movie of $\Sigma : \emptyset \to K$ to get a movie for $\overline{\Sigma} : K \to \emptyset$.

Choose some $\phi \in \mathrm{Kh}(K)$ and consider

$$\boxed{\Sigma_\phi := \left|\mathrm{Kh}(\overline{\Sigma})(\phi)\right| \in \mathbb{Z}}$$

which is an invariant of the bndry-preserving class of Σ (or $\overline{\Sigma}$)

Fixes many "complexity" issues:

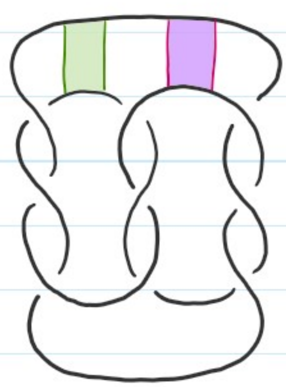
- by choosing $\phi \in \mathrm{Kh}(K)$ we can controle how awful calculations become
- can easily compare Σ_ϕ's (integers)

<u>Thm</u> The slice disks D_ℓ and D_r, given by band moves ℓ and r on the given knot K, are distinguished by the given class $\phi \in \mathrm{Kh}(K)$, $D_{\ell,\phi} \neq D_{r,\phi}$

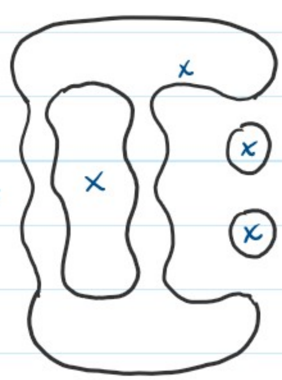
a. $K = 9_{46}$

$D_{\ell,\phi} = 0$

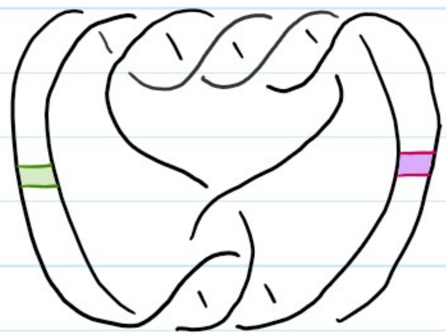
$D_{r,\phi} = 1$



$\phi =$



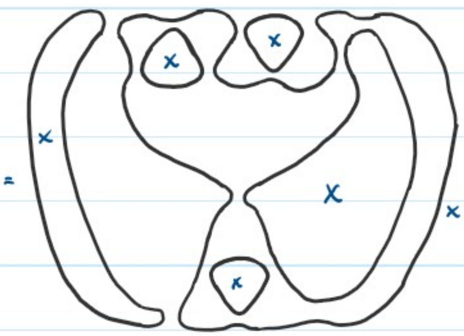
(b.) $K = 15n_{103488}$



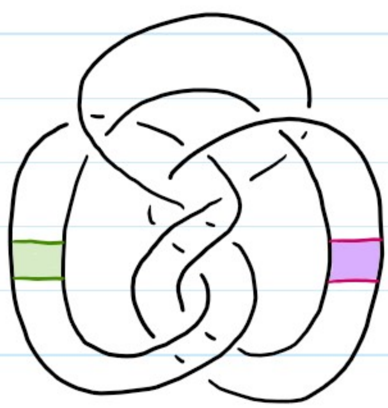
$D_{\ell,\phi} = 1$

$D_{r,\phi} = 0$

$\phi =$



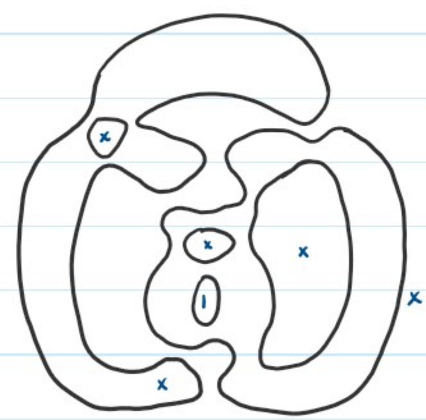
(c.) $J = 17nh_{34}$



$D_{\ell,\phi} = 1$

$D_{r,\phi} = 0$

$\phi =$



FACT: The slices D_ℓ and D_r for J are continuously isotopic rel J 

Since they are not smoothly isotopic rel J, we say they are exotic disks.

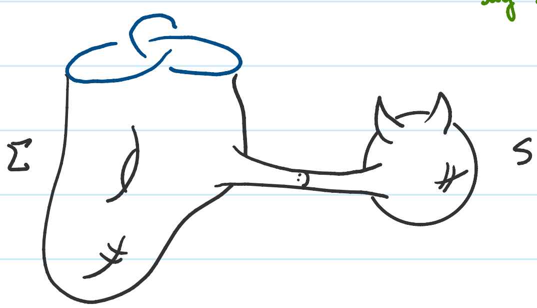
↳ Can be generalized to inf-many ex's with any genus that are not ambiently isotopic

used pairs of bands

extended by a ribbon concordance to (amphichiral) knot with trivial symmetry group

CHEATING Given a link cob. $\Sigma : L_0 \to L_1$ and a knotted 2-sphere S, the surfaces Σ and $\Sigma \# S$ are (generally) not isotopic

say Σ is locally knotted



$L_0 = \emptyset$

Thm (Hayden-S., Swann) The cobordism maps on Khov. hom. do not detect local knotting: $Kh(\Sigma) = \pm Kh(\Sigma \# S)$