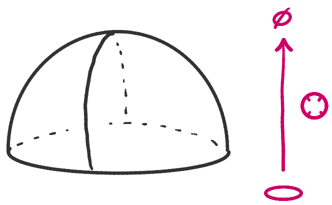
Talk: MPIM 2 (hand out)

September 9, 2021 1:35 PM

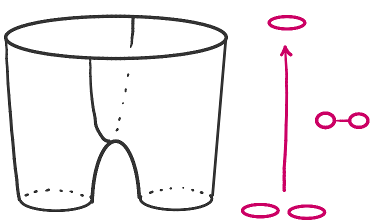
Here are the maps from last time, used to define our TQFT.
Note the new shorthand for these cobordisms.



$\iota : A \longrightarrow R$

$1 \longmapsto 0$

$x \longmapsto 1$



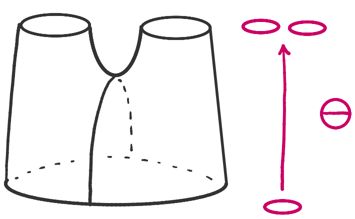
$m : A \otimes A \longrightarrow A$

$1 \otimes 1 \longmapsto 1$

$1 \otimes x \longmapsto x$

$x \otimes 1 \longmapsto x$

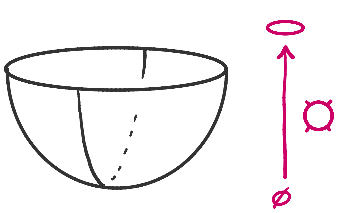
$x \otimes x \longmapsto 0$



$\Delta : A \longrightarrow A \otimes A$

$1 \longmapsto x \otimes 1 + 1 \otimes x$

$x \longmapsto x \otimes x$

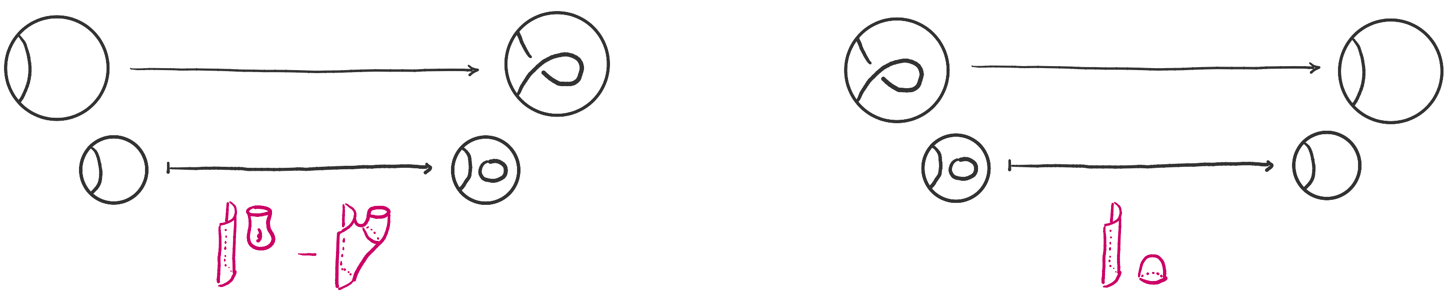


$\varepsilon : R \longrightarrow A$

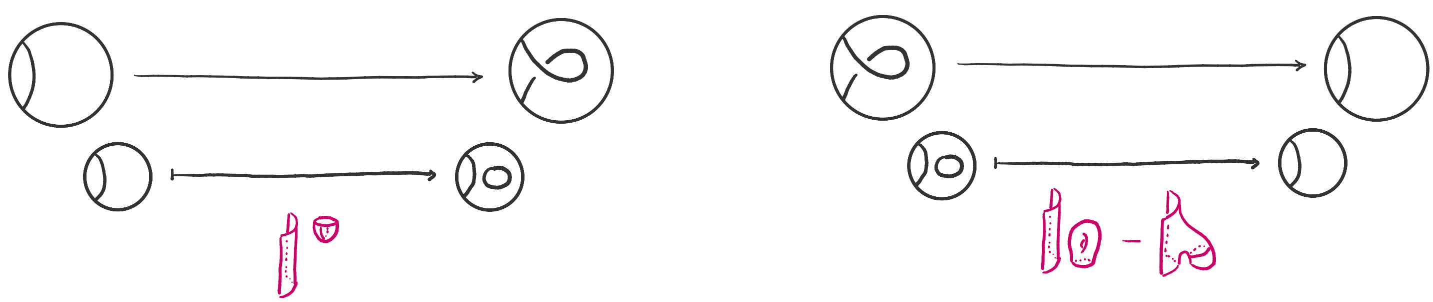
$1 \longmapsto 1$

Here are the Reidemeister I and II induced maps. We relate smoothings by linear combinations of cobordisms. Given a labeling, we apply the corresponding Morse induced map. Smoothings not shown are mapped trivially (to the "zero cobordism").

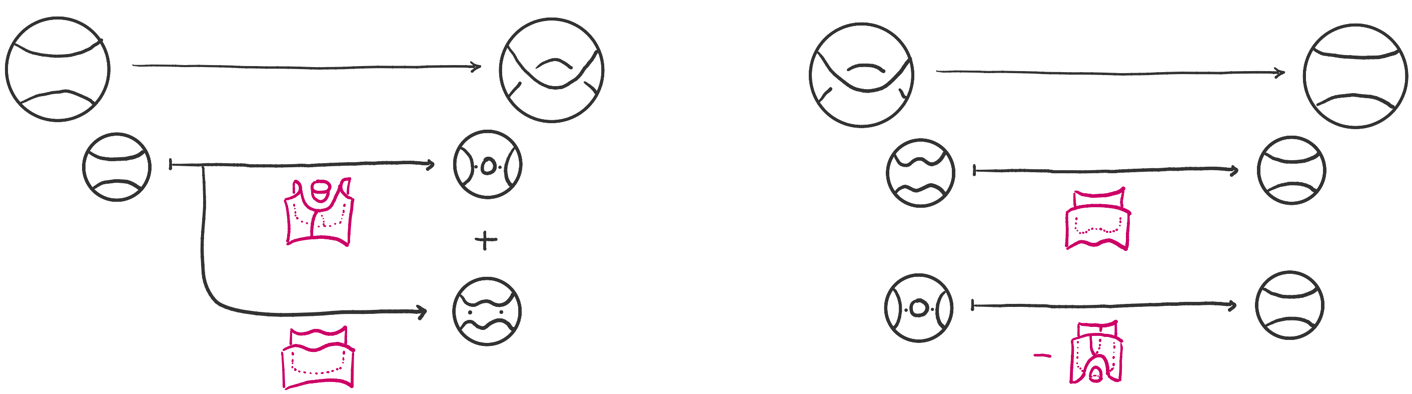
Positive RI



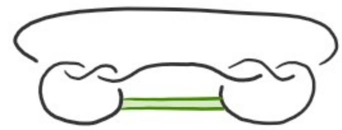
Negative RI



RII



<u>Exercise</u> Find $\phi \in Kh(3_1 \# m(3_1))$ so that the slice disk D illustrated below satisfies $Kh(D)(\phi) = 1$.



(a) Write out a movie for this slice disk

(b) Find a candidate cycle ϕ

i. What bigrading is $Kh(D)$ supported in?

ii. How many 0 and 1 smoothings does ϕ have?

iii. How many 1 and x labels can ϕ have?

iv. Make some guesses at ϕ

v. Make sure ϕ is a cycle $(\partial\phi = 0)$

(c) Show $Kh(D)(\phi) = \pm 1$.

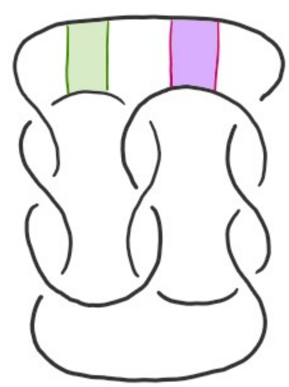
(d) <u>Bonus</u> Find a second class with this property.

<u>Exercise</u> Here are two slice disks D_ℓ and D_r for 9_{46}, given as band moves. Show that they induce distinct maps $Kh(D_{\ell,r}) : Kh(9_{46}) \longrightarrow Kh(\emptyset)$

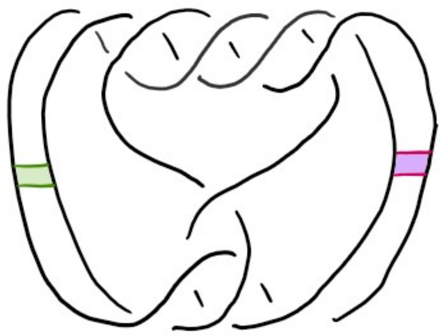
(a) Follow above steps to find $\phi \in Kh(9_{46})$

(b) Show $Kh(D_\ell)(\phi) = 0$ and $Kh(D_r)(\phi) = 1$

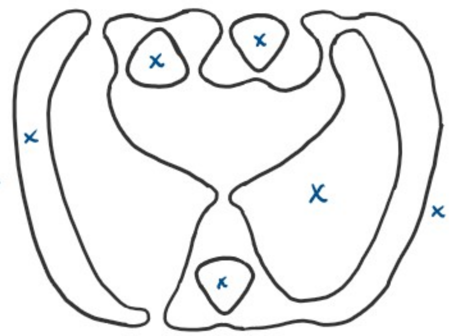
(c) What can we conclude about D_ℓ and D_r?

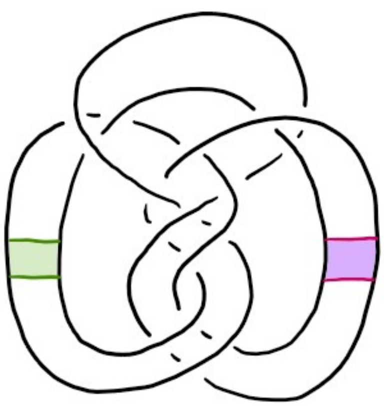


<u>Exercise</u> Show the surfaces below are distinct by distinguishing their induced maps on the given homology class.

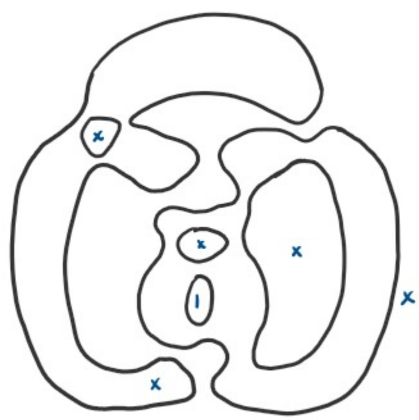


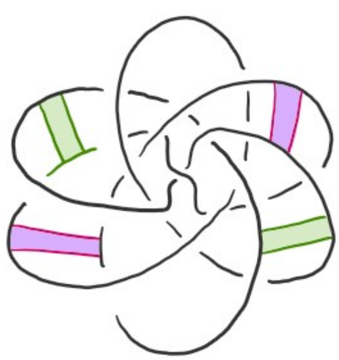
$\phi =$





$\phi =$





$\phi =$